Laminar Boundary-Layer Separation from an Upstream-Moving Wall

Demos T. Tsahalis*
Virginia Polytechnic Institute and State University, Blacksburg, Va.

The laminar boundary layer and separation for a steady outer inviscid flow over an upstream-moving wall are studied numerically for the first time. This is achieved by numerically integrating the unsteady boundary-layer equations for an impulsively started upstream-moving wall. An upwind differencing scheme permits the integration through the layer of reversed flow that covers the entire wall immediately after the impulsive start. The numerical integration is allowed to continue for large times until a steady-state solution is achieved. It is found that the point of separation, as defined by Sears and Telionis, starts moving upstream and arrives asymptotically at a station where all the properties of separation according to the model of Moore, Rott, and Sears are present.

I. Introduction

UNSTEADY viscous flows have been studied rather extensively and all the characteristic features of unsteady effects are now more or less familiar to fluid mechanicians. Stewartson,[1] Stuart,[2,3] and recently Telionis[4] have concisely reviewed the main ideas and important contributions on the topic.

One of the characteristic features of such flows is the phenomenon of separation. It should be mentioned that separation is defined here as the phenomenon of large-scale breaking away of fluid from the wall, which marks the breakdown of the boundary-layer approximation and the initiation of a wake. The location of actual separation has a drastic effect on control forces generated by aerodynamic surfaces and loads carried by aerodynamic structures and regulates phenomena like the stall flutter of an airfoil or the rotating stall of actual flow compressors.

It has long been recognized that Prandtl's classical separation criterion of vanishing skin friction may lead to erroneous results in cases other than two-dimensional flows over fixed walls. Sears[5] proposed a definition for separation in unsteady flow, namely, "the appearance of a stagnation point and a dividing flow trajectory between boundary layer and wake fluids in the flow seen by an observer moving with the separation point." For the case of steady flow over moving walls, Moore,[6] Rott,[7] and Sears[5] have suggested as criterion for separation the condition: $\partial u/\partial y=0$ at $u=0$ instead of $y=0$. Hereafter, this criterion, the point at which this criterion is met and the corresponding profile will be described by the abbreviation, "MRS". Finally, Sears and Telionis[8] have proposed a criterion for unsteady separation based on the concept of Goldstein's[9] singularity, which is considered to be the most reliable means for the boundary layer to signal the location of separation.

Sears[5] and Moore and Hartunian[6,10] have pointed out the similarities between unsteady flow over fixed walls and steady flow over moving walls. They demonstrated that steady separating flow over a downstream-moving wall is a case of unsteady flow with upstream-moving separation point, when viewed by an observer moving with the wall, and vice versa for an upstream-moving wall. Thus, elucidation of the phenomena of the latter case, which is the subject of the present paper, is important for understanding of the related unsteady case.

Vidal[11] and Ludwig[12] experimentally verified the MRS separation criterion for steady flow over moving walls. Telionis and Werle[13] and Tsahalis and Telionis[14] studied numerically the problem of steady separating flow over a downstream-moving wall. In these references, it was shown that a separation singularity appears at the MRS point instead of the point of zero skin friction. Williams and Johnson[15,16] transformed certain classes of semisimilar unsteady flows with upstream-moving separation to steady flows over a downstream-moving wall and verified the MRS criterion for separation, which was again found to be accompanied by a separation singularity. Fansler and Danberg[17] employed an integral method and calculated nonsimilar steady two-dimensional boundary-layer development over both upstream- and downstream-moving walls. They proposed a criterion for separation based on the minimum value of the shape factor and reported that a Goldstein-type singularity was encountered at separation.

Truly unsteady boundary layers corresponding to impulsive, transient, and oscillating flows, with upstream-moving or stationary separation, have been studied by Tsahalis and Telionis.[18-23] They demonstrated that the point of zero wall shear is nonsingular in unsteady flows and that breakdown of the boundary layer calculation occurs further downstream signaled by a singular behavior of the boundary layer solution. Then then interpreted this singular breakdown of the boundary-layer solution as unsteady separation, according to the theory of Sears and Telionis.[8] They overcame the difficulty of integrating the boundary-layer equations in the wrong direction, through regions of reversed flow, by employing an upwind differencing scheme that is unconditionally stable.[18,19,24]

To the author's knowledge there is no theoretical treatment valid near separation for steady flow over an upstream-moving wall; a numerical solution of this problem has not been achieved. The absence of a numerical solution is obviously due to the inability of integrating the steady boundary-layer equations through regions of noncirculating reversed flow. In the present paper the laminar boundary layer and separation of a steady flow over an upstream-moving wall is studied numerically for the first time. This is achieved by solving the unsteady boundary-layer equations for an impulsively started upstream-moving wall. The program is allowed to continue the numerical integration for large times until a steady-state solution is achieved. An upwind differencing scheme[19] is employed to integrate the boundary-layer equations through the layer of reversed flow which covers the entire wall for all times immediately after the

Presented as Paper 76-377 at the AIAA 9th Fluid and Plasma Dynamics Conference, San Diego, Calif., July 14-16, 1976; submitted Aug. 11, 1976; revision received Jan. 18, 1977.

Index categories: Boundary Layers and Convective Heat Transfer – Laminar; Nonsteady Aerodynamics.

*Research Associate, Engineering Science and Mechanics Department; now Research Engineer with Shell Development Company, Houston, Texas. Member AIAA.

impulsive start. Considering also the demonstrated strong resemblance between the features of separation in the cases of steady flow over moving walls and unsteady flow over fixed walls,[5,6,8,15,16] it is believed that the present numerical investigation also provides information about the features of a downstream-moving unsteady separation.

II. Governing Equations

Let U_∞ be the freestream velocity and L a typical length of the problem. If U_∞ and L are used to nondimensionalize distances, time, and velocities, the unsteady boundary-layer equations, for a steady outer flow, become

$$\frac{\partial u}{\partial s}+\frac{\partial v}{\partial N}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial s}+v\frac{\partial u}{\partial N}-U_e\frac{\partial U_e}{\partial s}=\frac{\partial^2 u}{\partial N^2} \tag{2}$$

where s and N are the coordinates and u and v are the velocity components along and perpendicular to the surface of the body, respectively, t is the time, $U_e(s)$ is the outer flow velocity and $u_w(s,t)$ is the velocity of the wall. The coordinate N and the velocity component v have been stretched with the square root of the Reynolds number, $Re=U_\infty L/\nu$, where ν is the kinematic viscosity of the fluid. The appropriate boundary and initial conditions are

$$u(s,0,t)=u_w(s,t),\ v(s,0,t)=0 \tag{3}$$

$$u(s,N,t)\rightarrow U_e(s) \text{ as } N\rightarrow\infty \tag{4}$$

and

$$u(s,N,0)=u_I(s,N) \tag{5}$$

$$v(s,N,0)=v_I(s,N) \tag{6}$$

respectively. In the present problem, the wall is started impulsively moving-upstream from rest, with a velocity $u_w(s)$.

A modification of Görtler's[25] transformation is now introduced with new independent variables

$$\xi=\int_0^s U_e(s)\mathrm{d}s \tag{7}$$

$$\eta=[U_e(s)/\sqrt{2\xi}]N \tag{8}$$

and new dependent variables

$$F=u/U_e \tag{9}$$

$$V=\frac{\sqrt{2\xi}}{U_e}v+\frac{2\xi}{U_e}\frac{\partial\eta}{\partial s}F \tag{10}$$

for the velocity components in the ξ and η directions, respectively.

In terms of the new dependent and independent variables, the continuity and momentum equations (1) and (2) become

$$2\xi\frac{\partial F}{\partial\xi}+F+\frac{\partial V}{\partial\eta}=0 \tag{11}$$

$$\frac{2\xi}{U_e^2}\frac{\partial F}{\partial t}+2\xi F\frac{\partial F}{\partial\xi}+V\frac{\partial F}{\partial\eta}+\beta(F^2-1)=\frac{\partial^2 F}{\partial\eta^2} \tag{12}$$

where β is the pressure gradient function given by

$$\beta=\frac{2\xi}{U_e}\frac{\partial U_e}{\partial\xi} \tag{13}$$

The boundary and initial conditions in terms of the new variables are

$$F(\xi,0,t)=F_w(\xi) \qquad V(\xi,0,t)=0 \tag{14}$$

$$F(\xi,\eta,t)\rightarrow 1 \text{ as } \eta\rightarrow\infty \tag{15}$$

and

$$F(\xi,\eta,0)=F_I(\xi,\eta) \tag{16}$$

$$V(\xi,\eta,t)=V_I(\xi,\eta) \tag{17}$$

III. Impulsively Started Motion of the Wall

In the present problem, the outer flow distribution was chosen to be the linearly decelerated distribution considered by Howarth[26]

$$U_e(s)=1-As, \qquad A=0.12 \tag{18}$$

At time $t=0$, the wall is started impulsively moving upstream from rest, with a velocity distribution given by

$$u_w(s)=-ACs \qquad AC>0 \tag{19}$$

The impulsively started motion of the wall generates vortex sheets that cover the entire wall and subsequently diffuse into the flow. Figure 1 schematically depicts this situation.

If $u_H(s,N)$ and $v_H(s,N)$ correspond to the steady-state solution with an outer flow given by Eq. (18) and a fixed wall, then at $t=0^+$ the flow field is given by

$$u(s,N,0^+)=u_H(s,N) \text{ for } N>0 \tag{20}$$

$$u(s,0,0^+)=-ACs \tag{21}$$

$$v(s,N,0^+)=v_H(s,N) \tag{22}$$

The flow field given by Eqs. (20)-(22) is considered as the initial condition of the unsteady problem.

For $t\leq 0$ the steady-state solution, $u_H(s,N), v_H(s,N)$, can be obtained by a steady-state scheme of integration and checked with Ref. 26. For this case, the point of separation is unambiguously defined as the point of zero wall shear. Howarth's[26] calculations have shown that the distance to separation is given by $S_{zf}\simeq 0.12/A$. In the present problem, $S_{zf}\simeq 1.0$ since $A=0.12$. For $t\rightarrow\infty$, the flow field corresponds to a steady-state solution with the same outer flow distribution and an upstream-moving wall with velocity distribution given by Eq. (19). Since separation in laminar flows is hastened when the wall is moving upstream, the corresponding separation point S_{MRS} is expected to be located upstream of S_{zf}; that is $S_{\mathrm{MRS}}<S_{zf}$. The distribution of the

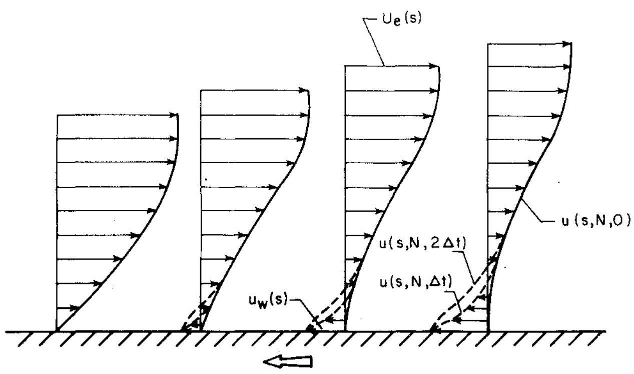

Fig. 1 Schematic of the impulsive start of the wall. The dotted lines denote the expected change for $t>0$ due to vorticity diffusion.

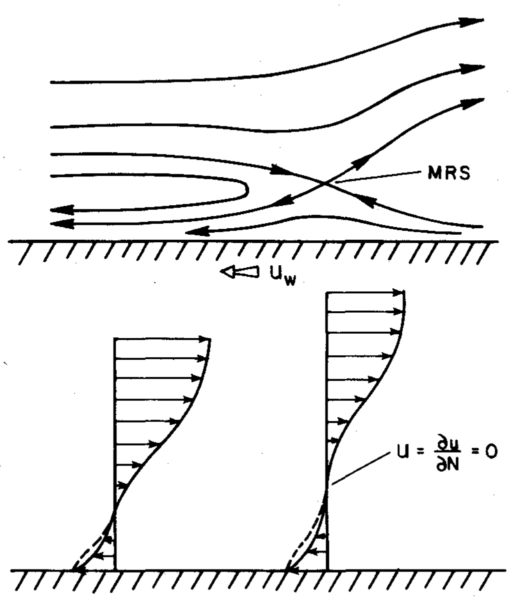

Fig. 2 Schematic sketch of the streamline pattern and boundary-layer velocity profiles near separation for an upstream-moving wall.

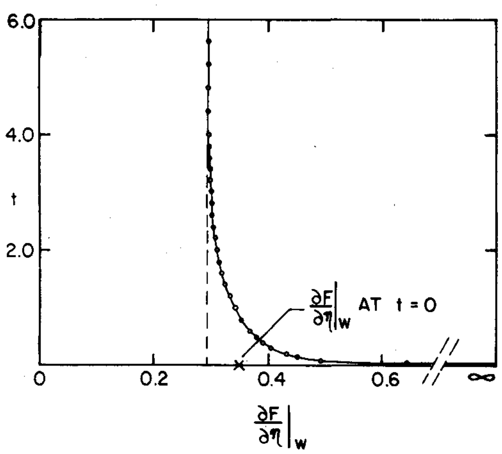

Fig. 3 The wall shear function at $s=0.4$ plotted against time for $AC=0.18$.

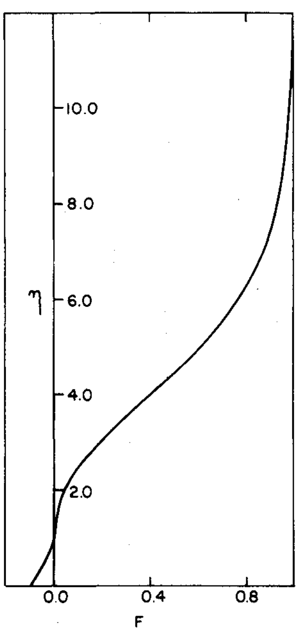

Fig. 4 Velocity profile, at $s=0.495$ and $t=8.0$, in the neighborhood of the MRS station for $AC=0.18$. The actual separation point is located between $s=0.495$ and $s=0.500$.

wall velocity adopted in the present problem has the advantage of introducing mildly the motion of wall and generating vortex sheets of decreasing strength as $s \to 0$ for $t=0^+$, thus eliminating possible numerical instabilities during integration in the neighborhood of $s=0$ and small times.

The numerical analysis encounters a difficulty for small times and very close to the wall. This difficulty is due to the inability of the numerical scheme to reproduce a vortex sheet. An approximate analytic solution was therefore derived and incorporated in the computer program to help start the integration. It is known that for impulsive changes and for very small times the convection terms of the momentum equation are much smaller than the unsteady and viscous terms. Therefore, the problem in the nondimensional form can be approximated with

$$\partial u_i / \partial t = \partial^2 u_i / \partial N^2 \tag{23}$$

$$u_i(s,N,0) = u_H(s,N) \tag{24}$$

$$u_i(s,0,t) = -ACs \tag{25}$$

$$u_i(s,N,t) \to U_e(s) \text{ as } N \to \infty \tag{26}$$

The solution to the above problem is

$$u_i(s,N,t) = u_H(s,N) - ACs \operatorname{erfc}\left(\frac{N}{2\sqrt{t}}\right) \tag{27}$$

where erfc is the complementary error function. The quantities $u_H(s,N)$, $v_H(s,N)$ corresponding to the steady-state solution were first calculated and stored. Then, $u_H(s,N)$ was replaced by $u_i(s,N,t)$ evaluated at a very small time and the unsteady computer program was left to continue the integration.

IV. The Numerical Integration

Writing the time derivative in a difference form as

$$\partial F / \partial t = [F(s,\eta,t) - F(s,\eta,t-\Delta t)] / \Delta t \tag{28}$$

and using the notation $F^\circ = F(s,\eta,t-\Delta t)$ the unsteady momentum equation, Equation (12), can be written as

$$\frac{\partial^2 F}{\partial \eta^2} - V\frac{\partial F}{\partial \eta} - 2\xi F \frac{\partial F}{\partial \xi} - \left(\beta F + \frac{2\xi}{U_e^2 \Delta t}\right) F$$
$$+ \left(\beta + \frac{2\xi F^\circ}{U_e^2 \Delta t}\right) = 0 \tag{29}$$

Equation (29) has now the form of the steady-state momentum equation. Considering that the continuity equation (11) has the same form in either steady or unsteady flow, Eqs. (11) and (29) were solved numerically by a subroutine for steady flow developed by Werle and Davis.[27]

In the layer of reversed flow that covers the entire wall, the negative sign of the u velocity component reverses the proper direction of integration because of the nonlinearity of the momentum equation. Integration through this region of reversed flow was achieved by employing an upwind differencing scheme.[19] According to this scheme the ξ derivative at the kth station in the ξ direction is replaced by

$$\frac{\partial F}{\partial \xi}\Big|_k = \frac{1}{2\Delta\xi}\left[(F^\circ_{k+1} - F^\circ_k) + (F_k - F_{k-1})\right] \tag{30}$$

The flow field $u_H(s,N)$, $v_H(s,N)$ with an outer flow distribution given by Eq. (18) and fixed wall was first calculated. Then $u_H(s,N)$ was replaced by $u_i(s,N,t)$ evaluated at a small time t_0 and the flow field $u_i(s,N,t_0)$,

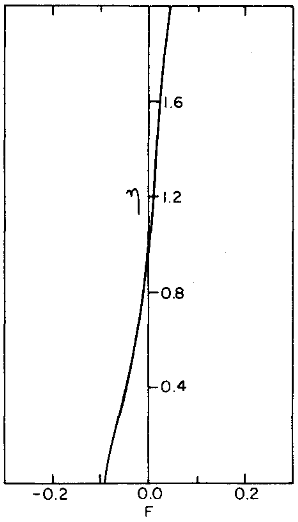

Fig. 5 Detail of the velocity profile of Fig. 4 near the wall.

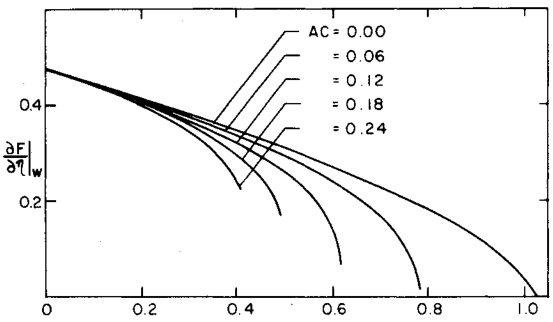

Fig. 6 The wall shear function vs the distance s for steady flow over fixed or upstream-moving wall.

$v_H(s,N)$ was stored to provide the initial conditions for integration with respect to time. Time was then incremented to $t_0+\Delta t$ and the (ξ,η) plane was swept again in the downstream direction using the data stored at $t=t_0$ to form derivatives according to Eqs. (28) and (30). The new data at time $t=t_0+\Delta t$ replaced those at time $t=t_0$ in the storage; time was then incremented again to $t=t_0+2\Delta t$ and this process was repeated for all subsequent time steps.

V. Results and Discussion

It was shown in Refs. 13, 14, 18, and 19, that in unsteady flow or steady flow over a downstream-moving wall the point of zero skin friction is not singular. Furthermore, it was demonstrated in these references that all the typical features of a Goldstein[9] singularity, that is, blowing up of quantities like $\partial^2u/\partial N\partial s$, v, δ^*, etc. with the inverse square root of the distance in the upstream direction from separation, appear at a point which Sears and Telionis[8] define as separation. The appearance of the Goldstein singularity has also been used in the present problem to define separation, according to the theory of Ref. 8.

At this point some comments are necessary concerning the MRS profile near separation in the case of an upstream-moving wall (see Fig. 2). It has been pointed out in Ref. 17 that for constant wall velocity as well as the wall velocity distribution considered in the present problem and adverse pressure gradient, the velocity profile curvature at the wall must be positive, a condition obviously not satisfied by the MRS profile. This correction, though, can be easily introduced into the MRS profile without altering its basic features or changing the associated streamline configuration. In Fig. 2 the proposed correction to the MRS profile, that meets the above compatibility condition, is shown with a broken line. It has also been argued in Ref. 17 that, since the MRS point is an inflection point, that is, $\partial^2u/\partial N^2=0$, and also $u=\partial u/\partial N=0$, the momentum equation is unbalanced at the MRS point for a nonzero pressure gradient. Therefore the MRS criterion cannot be applied in this case. This argument, however, is based on the assumption that the boundary-layer approximation is valid up to the separation point, a fact that is not true. Indeed, as the neighborhood of separation is approached, the boundary-layer approximation breaks down gradually and the term $\partial^2u/\partial s^2$, neglected during the formulation of the boundary-layer equation, becomes important and balances the pressure gradient term in the momentum equation. Furthermore, it is the omission of the term $\partial^2u/\partial s^2$ that makes the boundary-layer equations exhibit the well-established singular behavior near separation. In the light of the above comments, it is argued that the MRS criterion is valid for the case of an upstream-moving wall and that solutions of the boundary-layer equations can only approximate the MRS separation criterion as close as possible.

It has been pointed out in the introduction that the numerical integration was allowed to continue for large times until a steady-state solution was obtained. This fact can be detected by examining the variation of the wall shear at a fixed station. In Fig. 3, the quantity $\partial F/\partial\eta|_w$ at $s=0.4$ and $AC=0.18$, is plotted vs time. Immediately after the impulsive start $\partial F/\partial\eta|_w$ becomes infinite because of the generated vortex sheet. Then for small times $\partial F/\partial\eta|_w$ starts decreasing at a very large rate, characteristic of the diffusion of concentrated vorticity, and the typical features of the erfc function are still present. For times larger than 1, $\partial F/\partial\eta|_w$ decreases at a smaller rate and finally, for $t>4.0$, it tends asympotically to a constant value.

The location of the singularity, which for $t=0$ coincides with the separation point S_{zf} over fixed wall, appeared to be stationary for small times. For larger times, it started traveling upstream and finally arrived asymptotically at a fixed location. This traveling singularity was interpreted as unsteady separation, and its fixed location for $t\to\infty$ was interpreted as the separation point S_{MRS} for steady flow over an upstream-moving wall. The properties of the steady flow field, obtained for $t\to\infty$, that correspond to a steady flow over an an upstream-moving wall are discussed below.

In Fig. 4 the velocity profile $F(\eta)$, for $AC=0.18$, has been plotted near the separation point S_{MRS}. A detail of the same profile near the wall has been plotted in Fig. 5. It is clear that this velocity profile has just the shape of the corrected MRS velocity profile and closely approximates the MRS separation

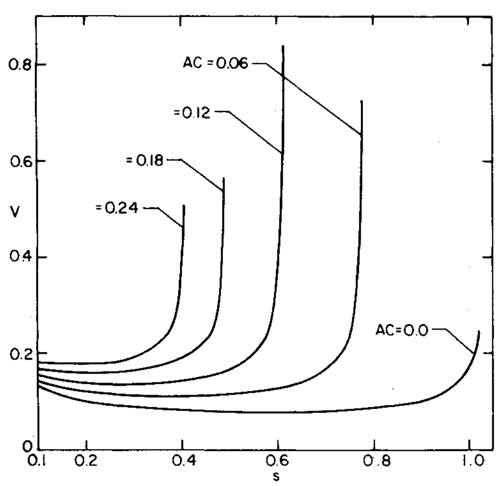

Fig. 7 The vertical velocity component v at $\eta=0.4778$ vs the distance s for steady flow over fixed or upstream-moving wall.

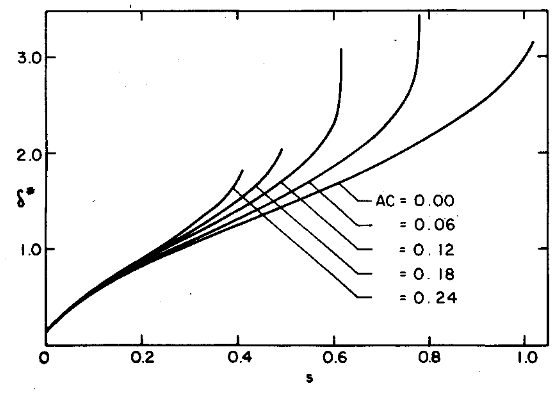

Fig. 8 **The displacement thickness δ^* vs the distance s for steady flow over fixed or upstream-moving wall.**

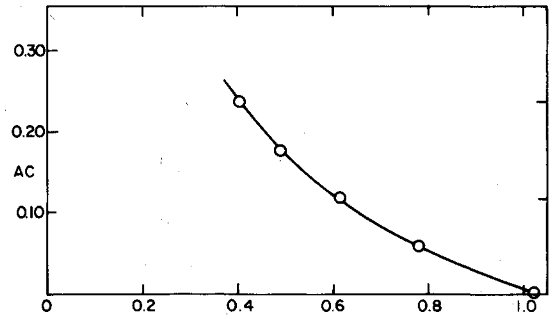

Fig. 9 **The location of the MRS separation point for different wall velocities.**

criterion (notice the flattening of the velocity profile in the neighborhood of $F \simeq 0$).

In Fig. 6, the wall shear $\partial F/\partial\eta|_w$ has been plotted vs the distance s for different AC's. Notice the steepening of the slope $\partial(\partial F/\partial\eta|_w)/\partial s$, which finally becomes infinite as the point of separation is approached, indicative of singular behavior.

In Fig. 7, the normal velocity component v, at a fixed distance from the wall, has been plotted as a function of the distance s for different values of AC. Notice the singular behavior (blowing up) of v near the point of separation, which is in qualitative agreement with the experimental results of Ludwig[12] who concluded that, "Indeed, the only reliable experimental indication of separated flow when the wall is moving upstream appears to be the behavior of the vertical velocity component in the boundary layer."

In Fig. 8 the displacement thickness δ^* is shown plotted versus the distance s for different AC's. Notice again the abrupt increase of δ^* as the separation point is approached.

From Fig. 6-8, it is deduced that the MRS station, which is achieved asymptotically as shown in Fig. 4, coincides in the boundary-layer model with a Goldstein-type singularity. This station was shown experimentally[12] to represent separation.

In Fig. 9, the location of the point of separation has been plotted for different values of AC.

VI. Conclusions

The laminar boundary layer and separation for a steady outer inviscid flow over an upstream-moving wall have been studied numerically. It has been demonstrated that a Goldstein-type singularity appears again as in the case of steady flow over fixed or downstream-moving walls. The present results strongly suggest that the station, where the Goldstein singularity appears, coincides with the station of separation for upstream-moving walls, in agreement with the experimental results of Refs. 11 and 12. It has also been shown that the MRS criterion is approximately met at this station.

In conclusion, the present results have demonstrated that the MRS separation criterion, for the case of a steady flow over an upstream-moving wall, is valid and separation coincides in the boundary-layer model with a Goldstein-type singularity. Considering also the demonstrated strong resemblance between the features of separation in steady flow over a moving wall and unsteady flow over fixed walls, it is believed that the present results provide support to the theoretical model of Ref. 8 for the case of a downstream-moving unsteady separation.

Acknowledgment

Research sponsored by the Air Force Office of Scientific Research, Air Force Systems Command, U.S. Air Force, under Grant AFOSR-74-2651A. The author is thankful to Professors W.R. Sears and D.P. Telionis for stimulating discussions.

References

[1]Stewartson, K., "The Theory of Unsteady Laminar Boundary Layers," *Advances in Applied Mechanics*, Vol. 6, Academic Press, New York, 1960, pp. 1-37.

[2]Stuart, J.T., "Unsteady Boundary Layers," *Laminar Boundary Layers*, Oxford Univ. Press, London, 1964, pp. 349-406.

[3]Stuart, J.T., "Unsteady Boundary Layers," *Recent Research on Unsteady Boundary Layer, Proceedings of the Symposium of International Union of Theoretical and Applied Mechanics*, Laval Univ. Press, Quebec, May 1971.

[4]Telionis, D.P., "Calculations of Time Dependent Boundary Layers," in *Unsteady Aerodynamics*, R.B. Kinney (ed.), Vol. 1, July 1975, pp. 155-190.

[5]Sears, W.R., "Some Recent Developments in Airfoil Theory," *Journal of the Aeronautical Sciences*, Vol. 23, 1956, pp. 490-499.

[6]Moore, F.K., "On the Separation of the Unsteady Laminar Boundary Layer," in *Boundary Layer Research, Proceedings of the Symposium of the International Union of Theoretical and Applied Mech.*, H. Gortler (ed.), 1957, pp. 291-311.

[7]Rott, N., "Unsteady Viscous Flow in the Vicinity of a Stagnation Point," *Quarterly Journal of Applied Mathematics*, Vol. 13, 1956, pp. 444-451.

[8]Sears, W.R. and Telionis, D.P., "Unsteady Boundary-Layer Separation," in *Recent Research of Unsteady Boundary Layers*, E.A. Eichelbrenner (ed.), Vol. 1, 1971, pp. 404-447.

[9]Goldstein, S., "On Laminar Boundary-Layer Flow over a Position of Separation," *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 1, 1948, pp. 43-69.

[10]Hartunian, R.A. and Moore, F.K., "Research on Rotating Stall in Axial-Flow Compressors, Part II – On the Separation of the Unsteady Laminar Boundary Layer," WADC TR-59-72, 1959.

[11]Vidal, R.J., "Research on Rotating Stall in Axial-Flow Compressors, Part III – Experiments on Laminar Separation From a Moving Wall," WADC TR-59-75, Jan. 1959.

[12]Ludwig, G.R., "An Experimental Investigation of Laminar Separation from a Moving Wall," AIAA Paper 64-6, N.Y., Jan. 1964.

[13]Telionis, D.P. and Werle, M.J., "Boundary Layer Separation from Moving Boundaries," *Journal of Applied Mechanics*, Vol. 95, 1973, pp. 369-374.

[14]Tsahalis, D. T. and Telionis, D.P., "The Effect of Blowing on Laminar Separation," *Journal of Applied Mechanics*, Vol. 40, Dec. 1973, pp. 1133-1134.

[15]Williams, J.C., III, and Johnson, W.D., "Semi-Similar Solutions to Unsteady Boundary-Layer Flows Including Separation," *AIAA Journal*, Vol. 12, Oct. 1974, pp. 1388-1393.

[16]Williams, J.C., III, and Johnson, W.C., "Note on Unsteady Boundary-Layer Separation," *AIAA Journal*, Vol. 12, Oct. 1974, pp. 1427-1429.

[17]Fansler, K.S. and Danberg, J.E., "An Integral Analysis of Boundary Layers on Moving Walls," Ballistic Research Laboratories, Aberdeen Proving Ground, Md., Report No. 1792, June 1975.

[18]Tsahalis, D. T., "The Boundary-Layer Separation Singularity in Steady and Unsteady Flows," Master's Thesis, Virginia Polytechnic Institute and State University, Blacksburg, Va. 1972.

[19]Tsahalis, D.T., "Unsteady Boundary Layers and Separation," Ph.D. Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, Va. 1974.

[20]Telionis, D.P. and Tsahalis, D.T., "The Response of Unsteady Boundary-Layer Separation to Impulsive Changes of Outer Flow," *AIAA Journal*, Vol. 12, May 1974, pp. 614-619.

[21]Telionis, D.P. and Tsahalis, D.T., "Unsteady Laminar Separation over Impulsively Moved Cylinders," *Acta Astronautica*, Vol. 1, 1974, pp. 1487-1505.

[22]Tsahalis, D.T. and Telionis, D.P., "Oscillating Laminar Boundary Layers and Unsteady Separation," *AIAA Journal*, Vol. 12, 1974, pp. 1469-1476.

[23]Tsahalis, D. T. and Telionis, D.P., "Oscillating Boundary Layers with Large Amplitude," in *Unsteady Flows in Jet Engines*, F.O. Carta (ed.), 1974.

[24]Telionis, D.P., Tsahalis, D.T., and Werle, M.J., "Numerical Investigation of Unsteady Boundary-Layer Separation," *Physics of Fluids*, Vol. 16, Aug. 1973, pp. 968-973.

[25]Görtler, H., "A New Series for the Calculation of Steady Laminar Boundary-Layer Flows" *Journal of Mathematics and Mechanics*, Vol. 6, 1957, pp. 1-66.

[26]Howarth, L., "On the Solution of the Laminar Boundary-Layer Equations," *Proceedings of the Royal Society* London, Vol. A 164, 1938, pp. 547-579.

[27]Werle, M.J. and Davis, R.T., "Incompressible Laminar Boundary Layers on a Parabola at an Angle of Attack: A Study of the Separation Point," *Journal of Applied Mechanics*, Vol. 39, 1972, pp. 7-12.